Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Linear Algebra-I

Subject Code : 4SC03MTC2 Branch :B.SC (Mathematics)

Semester :3 Date : 26/04/2016 Time : 2:30 To 5:30 Marks :70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) True/false: Direct sum of two subspace is also subspace . (1)
b) What are the standard basis of R^2 ? (1)
c) Are (1,2,5) and (-5,-10,-25) linearly dependent ? (1)
d) What is inner product of (2,-2,7) and (0.5, -0.5, 0) . (1)
e) Find norm of (1,-5,0.2) . (1)
f) True/false: Every subspace is a vector space. (1)
g) Define span of {u,v} . (1)
h) Write dimension of M_{33}. (1)
i) What is dimension of C[0,1]? (1)
j) Find the angle between (1, 2 , 0 , 5) and (-2 ,1 ,5 ,0) . (1)
k) True/false: Every inner product space is norm linear space. (1)
l) Find the angle between (1, 2, 0) and (-2 , 1 ,5) . (1)
m) Define :Rank of linear transformation. (1)
n) True/false: M_{nn} is a vector space. (1)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Which of the following are subspace of V . **(8)**

(1) W=(x , y , z) / $x \geq 0$ } V=R3 .
(2) W=(x , y , z) / $x + y = 0$ } V=R3 .
(3) W=(x , y , z) / $xz \leq 0$ } V=R3 .
(4) W=(x , y , z) / $x - y + 2z = 0$ } V=R3 .

b) Define vector space and show that M22 is a vector space . **(6)**

Q-3 Attempt all questions **(14)**

a) Fix x0 ∈ X .Let S={ f:X→R / f(x0)=0} .Then Show that S is vector subspace of F(X,R). **(6)**

b) State and prove rank -nullity theorem. **(8)**

Q-4 **Attempt all questions** **(14)**

a) Check which of the following are L.T.? **(6)**

(1) T: R2→R3 , T(x , y)= (x, y, x + y)

(2) T: R3→R4 , T(x ,y ,z)=(x + y , y + z , z – y , x-y)

b) Check whether (1,2,4) ∈ span A **(6)**
Where A={ (0,1,-1) ,(0,0,2) ,(1,3,0) } .

c) Define inner product space. **(2)**

Q-5 Attempt all questions **(14)**

a) Examine the sub sets of R3 are L.D. or L.I. **(6)**
{ (1,2,1) ,(-1,3,0) ,(5,-2,9)

b) Prove that x is orthogonal to y if and only if $\|x+y\|^2=\|x\|^2+\|y\|^2$. **(4)**

c) Examine the sub set of C[0,2π] are L.D. or L.I. **(4)**
{ -sinx , -cosx , e^x }

Q-6 **Attempt all questions** **(14)**

a) Define subspace of vector space. let V is vector space W⊂ V.then show that W is subspace of V if and only if α u+ βv ∈ W for all α , β ∈ R and u,v ∈ W . **(8)**

b) State and prove Cauchy- Swarz inequality. **(6)**

Q-7 **Attempt all questions** **(14)**

a) Verify rank nullity theorem for T:R^4→R^2 such that **(6)**
$T(x_1,x_2,x_3,x_4)=(x_1-x_2+x_3-x_4,\ 2x_1+x_2+3x_3+x_4)$

b) Let V be a vector space and W_1 and W_2 be subspaces of V. prove that **(6)**
$\dim(W_1+W_2)=\dim W_1+\dim W_2-\dim(W_1\cap W_2)$.

c) Define inner product in C[0,1] also justify it. **(2)**

Q-8 **Attempt all questions** **(14)**

a) If V is vector space and W_1 , W_2 are two subspace of V then show that **(6)**

$W_1\cap W_2$ and W_1+W_2 are also subspace of V.

b) If V be inner product space .if we define d(x,y) =$\|x-y\|$, then show that d is a metric on V. **(6)**

c) What is Euclidian norm ? **(2)**